Professur für Mechanik auf Höchstleistungsrechnern
TUM School of Engineering and Design
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Master Thesis:

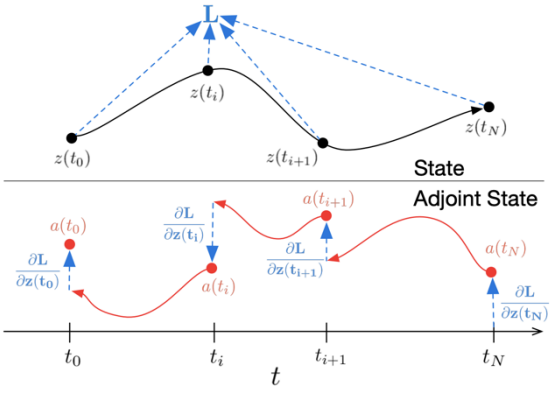
Adjoint Methods for Nonlinear Dynamic Problems: Expanded vs. Non-Expanded Formulations

Background and Motivation:

Inverse problems in nonlinear structural and cardiac mechanics require efficient and accurate gradient computations. For time-dependent problems, adjoint methods become challenging due to the path dependence introduced by time integration schemes such as generalized-α.

A common approach in the literature expands the system by including time-integration variables, leading to a larger coupled system (≈ 3× number of degrees of freedom) [1,2]. While this avoids complex recursive derivatives, it requires specialized solvers and preconditioning strategies.

An alternative approach avoids this expansion by analytically resolving the recursion and keeping the problem size unchanged. The relative advantages of these two fundamentally different strategies are currently not well understood.



Objective:

The objective of this thesis is to implement and analyze the expanded adjoint formulation for nonlinear dynamic problems based on generalized-α time integration, and to systematically compare it with an existing non-expanded (same-size) adjoint approach already developed within the research group. Building on the group's prior work and available implementation of the non-expanded method, the student will focus on implementing the expanded formulation from the literature, optimizing both approaches where necessary, and evaluating their performance in terms of accuracy, computational efficiency, numerical robustness, and solver and preconditioning requirements.

Although the topic is methodologically challenging, the work will be closely supervised and supported.

Tasks:

Literature review

- Study adjoint methods for time-dependent nonlinear problems
- Understand expanded (augmented) vs. non-expanded formulations

Implementation of expanded formulation

- Implement the adjoint system with expanded unknowns (~3× ndof)
- Assemble the corresponding linear systems

Solver and preconditioning strategy

- Identify and test suitable **iterative solvers**
- Design and evaluate **preconditioning strategies** for the expanded system

Comparison study

- Compare expanded vs. non-expanded approaches with respect to:
  - computational cost
  - implementation complexity

Analysis and conclusions

- Identify the advantages and limitations of both approaches
- Provide recommendations for practical applications

Qualifications:

- Strong background in applied mathematics, numerical methods, or computational mechanics
- Solid understanding of finite element methods and linear algebra
- Basic knowledge and interest in iterative solvers and preconditioning
- Very good programming skills in C++
- Interest in advanced topics such as adjoint methods and inverse problems

Supervisors: Prof. Dr.-Ing. Michael Gee; Tahar Arjoune, M.Sc.

[1] Alberdi R, Zhang G, Li L et al (2018) A unified framework for nonlinear path-dependent sensitivity analysis in topology optimization. Int J Numer Meth Eng 115(1):1–56.

[2] Arjoune, T., Bilas, C., Meierhofer, C. *et al.* Inverse analysis of patient-specific parameters of a 3D–0D closed-loop cardiovascular model with an exemplary application to an adult tetralogy of Fallot case. *Biomech Model Mechanobiol* 24, 2039–2068 (2025).